Performance Analysis of M-ary PSK Adaptive Modulation System over Rayleigh-Lognormal Fading Channel

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Abstract—An analytic study of applying M-ary PSK adaptive modulation scheme in Rayleigh-Lognormal fading channel is proposed. The close-form expression of the average BER and the average data throughput (BPS) of a proposed adaptive modulation system with constant transmitted power over fading channel are presented. The effect of shadowing on BER and throughput in adaptive modulation system is also investigated. The results show the analytical expressions of the average BER and data throughput derived for transmission over Rayleigh-Lognormal fading channel. Compare to the Mote Carlo simulation method, our analyzed results show good agreement to the simulation ones. The proposed method is useful in enhancing the data throughput of a communication system, such as third generation wireless communication system, where a huge amount of bandwidth is needed for high-speed multimedia services.

Keywords—Adaptive M-ary PSK; Rayleigh-Lognormal fading channel.

I. INTRODUCTION

Requirement for high quality of service (QoS) and high-speed multimedia transmission are rapidly increasing in the wireless systems. However, the system performance is strongly dependent on wireless channels, which vary over time due to multipath fading. Adaptive modulation (AM), or link adaptation, is an excellent technique to exploit these fluctuations to maximize the data throughput over such channels in energy and spectral efficient manners. In an AM scheme, many parameters can be adjusted according to the channel variations such as the transmit power, modulation level, symbol rate, and coding rate etc. Nowadays, many well-known wireless systems have involved the link adaptation strategy by using various combinations of coding rate and modulation level in their physical layer standards. For example, the Enhanced Data Services for GSM Evolution (EDGE) which define nine different coding and modulation pairs, each of them correspond to the suitable signal to noise ratio (SNR) range [1]. Furthermore, the HIPERLAN/2 and WLAN IEEE 802.11a standards also have provided the similar viewpoint to mitigate the effects of the frequency-selective fading among the neighboring sub-carriers [2, 3]. Traditionally, the analysis of the AM mainly focused on the flat fading channels [4-6]. In [4], AM technique is analyzed for Rayleigh and Nakagami fading channel assuming the perfect power control. However, the power control error (PCE), about 2 to 7 dB, is not negligible when the power control algorithm is not perfect [7]. This PCE is directly related to the strength of fading signals due to shadowing effects. In this paper, a performance analysis of the M-ary PSK AM in the Rayleigh-Lognormal fading channel is provided. The average data throughput and bit error rate (BER) of M-ary PSK AM are expressed. The numerical and simulated results then are compared in the Rayleigh-Lognormal fading channel with various PCE values to show the effects of PCE in M-ary PSK AM system.

II. SYSTEM MODEL

In this paper, we assume the perfect knowledge of the feedback channel state information (CSI) that is provided at the transmitter and have no time delay. The signal envelopes of the receiver are detected through coherent detection. Furthermore, the adaptive modulation, M-ary PSK schemes with different modes are provided at the transmitter. For each transmission with the perfect channel estimation assumption, the mode was adjusted to maximize the data throughput under average BER constrain, based on the instantaneous SNR.

Figure 1. Block diagram of the proposed adaptive modulation system
Figure 1 shows the system block diagram. The data stream \( b \) is adjusted to different constellation sizes \( M(\gamma) \) according to the previous instantaneous SNR, while \( x \) is the modulated symbol stream that corresponds to data stream and constellation size selection. Here, \( h \) is the fading channel gain and \( w \) is a complex Gaussian noise (zero-mean, time-invariant variance \( \sigma^2 \)). According to perfection channel estimation, the received signal envelope \( y \) can be accurately separated the estimated instantaneous SNR \( \gamma \) and fading channel envelope \( \hat{h} \), respectively. The estimated instantaneous SNR \( \gamma \) was returned to the transmitter to decide the next transmission modulation format. Moreover, \( \hat{h} \) was fed into the detection block to get the detected data stream \( \hat{b} \). Finally, the Monte Carlo simulation method is used to calculate the average data throughput and BER performances of an M-ary PSK AM system.

III. M-ARY PSK ADAPTIVE MODULATION OVER RAYLIGHEH-LOGNORMAL FADING CHANNEL

In this section, the closed-form expression of the average data throughput and BER performances of an M-ary PSK AM system over arbitrary specified fading channel are derived. Besides, an optimum selection procedure to decide the thresholds of the SNR partition set is presented.

A. Closed-Form Expression over the Arbitrary Fading Channel

Let \( h \) be the channel envelope, \( \gamma = h^* \cdot P_s / P_n \) be the instantaneous received signal to noise ratio (SNR), and \( \Gamma = h^* \cdot P_s / P_n \) be the average SNR of the channel at the input of the transmitter, where \( P_s = P \log_2 M \) is the average transmit power per symbol (\( P_s \) the average transmit power per bit and \( M \) the constellation size) and \( P_n \) is the power of noise. Let \( f_\gamma (\gamma) \) and \( F_\gamma (\gamma) \) be the probability density and cumulative density functions of \( \gamma \). Define \( K \) partitions for \( \gamma \) such that if \( A_i \leq \gamma < A_{i+1}, \quad i = 0, ..., K - 1 \), then AM system is said to be in the mode \( i \). \( A = \{A_i \} \) is the set of the thresholds of the partition. Therefore, the average data throughput \( D(\Gamma, A) \) can be described as

\[
D(\Gamma, A) = \sum_{i=0}^{K-1} \log_2 M(A_i) \int_{A_i}^{A_{i+1}} f_\gamma (\gamma) d\gamma .
\]  

(1)

The average BER performance of AM can be calculated as

\[
P_{av}(\Gamma, A) = \frac{1}{D(\Gamma, A)} \sum_{i=0}^{K-1} \log_2 M(A_i) P(\Gamma, A_i)
\]  

(2)

where \( P(\Gamma, A) \) is the average BER in the specific mode \( i \), given by,

\[
P(\Gamma, A_i) = \int_{A_i}^{A_{i+1}} P_{st}(\gamma) f_\gamma (\gamma) d\gamma
\]  

(3)

where \( P_{st}(\gamma) \) is the BER of the M-ary PSK in the additive white Gaussian noise (AWGN) channel.

For coherent detection, the average BER of a Gray-coded M-ary PSK for transmission in AWGN channel can be described as

\[
P_{st}(\gamma) = \frac{2}{\log_2 M(A)} Q\left( \sqrt{2\gamma \log_2 M(A)} \sin \left( \frac{\pi}{M(A)} \right) \right)
\]  

(4)

\[
P(\Gamma, A) = \Delta_i Q\left( \sqrt{\delta_i^2} \gamma \right), \quad \text{for } M(A) \geq 4
\]

where \( Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left( -\alpha^2 / 2 \right) d\alpha \), \( \Delta_i = 2 / \log_2 M(A) \) and \( \delta_i = 2 \log_2 M(A) \left| \sin(\pi / M(A)) \right| \right\} \). By changing the order of integration, Eq. (3) for the M-ary PSK AM can be expressed as:

\[
P(\Gamma, A) = \Delta_i F_i(\gamma) Q(\sqrt{\delta_i^2} \gamma) - F_i(\gamma) Q(\sqrt{\delta_i^2}) + \Psi
\]  

(5)

where

\[
\Psi = \int_{\delta_i}^{\infty} F_i(\gamma) \frac{1}{\sqrt{2\pi}} \exp\left( -\gamma^2 / 2 \right) d\gamma .
\]  

(6)

The analytical expressions are provided for the Gray-coded BER of each mode, used in equation (5) and (6), for arbitrarily statistical channel distribution. Once the cdf of the SNR and the set of the thresholds \( A = \{A_i \} \) are known, the average data throughput and error probability can be evaluated for an M-ary PSK AM system over arbitrarily statistical fading channels.

B. Optimal Mode Adaptation

As stated above, the transmission data rate is assigned to each threshold region. The threshold regions will be adjusted to maximize the data throughput, according to certain constraints. The average BER constraint can be expressed as

\[
P_{av}(\Gamma, A) = P_{av}.
\]  

(7)

Therefore, the Lagrangian function was from by optimize the set of SNR threshold. It is given by

\[
J(A, A, ..., A, A) = \sum \log_2 M(A) \int_{A_i}^{A_{i+1}} f_\gamma (\gamma) d\gamma
\]  

\[
+ \lambda_\Delta \sum \log_2 M(A) \int_{A_i}^{A_{i+1}} [P_{st}(\gamma) - P_{av}(\gamma)] d\gamma
\]

(8)

where \( \lambda_\Delta \) is the Lagrange multiplier. The optimal set of SNR thresholds are found through solving

\[
\frac{\partial J}{\partial A_i} = 0, \quad 0 \leq i \leq K - 1
\]

(9)

which result in

\[
\log_2 M(A_i) P_{st}(\gamma) - \log_2 M(A) P_{av}(\gamma) \frac{1}{\lambda} = - \left( 1 - \log_2 M(A_i) - \log_2 M(A) \right).
\]  

(10)
By eliminating the Lagrange multiplier $\lambda$, which is proposed in [8], the relationship between the SNR thresholds is given as:

$$
\log M(A)P_m,\hat{(A)} - \log M(A),\hat{(A)} = P_i(A), \ i = 2,3,...,K-1. \qquad(11)
$$

For K-mode adaptive modulation system, K-1 SNR thresholds ($A_0 = 0$ and $A_K = \infty$) need to be found. Therefore, it only needs K-1 dependent roots in the Eq. (10). Finding out the optimal set of SNR thresholds is equivalent to solve the roots of the Eq. (10) with K-1 dependent equations. Eventually, there are only K-2 equations in the Eq. (11). In the proposed method, a certain SNR threshold $A_i$ is initially assigned, which is the required SNR value under the BER constraint of the QPSK modulation scheme. Then, the BER approximation of M-ary PSK with Gray-encoded as a function of $\theta(t)$ is a stochastic process with a uniform distributions in the interval $[0,2\pi]$, and $w(t)$ is a zero-mean complex Gaussian background noise with parameter $\sigma^2_w$. The received signals in a narrowband propagation channel are assumed to be associated with complex random process, $Z(t)$, and

$$
Z(t) = a(t) + j b(t) = h(t)e^{j\theta(t)}. \qquad(15)
$$

Thus, the received fading envelope in a narrowband propagation channel at certain time interval can be expressed as

$$
|Z| = r = RL \qquad(16)
$$

Therefore, the mixture probability density function (pdf) of the received signal envelope in Rayleigh-Lognormal fading channel can be given as follows.

$$
f(r) = \int_0^\infty f_{x_2}(r)f_x(L)\,dL \qquad(17)
$$

where $f_{x_2}(r)$ is the conditional joint lognormal and Rayleigh pdf, given by,

$$
f_{x_2}(r) = \frac{r}{(\sigma_2^2L^2)}\exp \left(-\frac{r^2}{2(\sigma_2^2L^2)} \right) \qquad(18)
$$

The random variable, $L$, for shadowing is lognormally distributed with probability density function,

$$
f_L = \frac{1}{\sqrt{2\pi}\sigma_L} L \exp \left(-\frac{1}{2} \left( \ln(L) - \frac{\phi}{\sigma} \right)^2 \right) \qquad(19)
$$

where $\phi = (\ln 10)/20$, and $u_i$ and $\sigma_i$ are the mean and variance, respectively, of the associated Gaussian distribution (dB unit). Substituting Eqs. (18) and (19) into Eq. (17) yields a mixed Rayleigh-lognormal pdf that represents combined shadowing and fast fading of the received envelopes in a narrowband propagation channel.

### C. Rayleigh-Lognormal Fading Channel

The Rayleigh-Lognormal fading channel was first proposed by Suzuki to describe the statistical model for urban environments due to cluster (with reflections and/or diffractions etc) and local scattering effects [10]. Furthermore, a statistical model for the land mobile satellite channel dealing with both shadowing and multipath effects was presented by Loo [11]. The model assumes that the signal variation due to attenuation of the line-of-sight is slow fading (lognormal) and that the variation due to multipath is fast fading (Rayleigh). Recently, many studies proposed a Rice-lognormal channel model by considering the received signal envelope $r$ as the product of two independent processes, $R$ and $L$ [12, 13]. The Suzuki and Loo models can be treated as a special case of the Rice-lognormal channel model.

The received signal envelopes of a narrowband propagation channel can be modeled as a random process. The received signal is given by

$$
y(t) = h(t)x(t - \tau)e^{j\theta(t)} + w(t) \qquad(14)
$$

where $x(t)$ is the transmitted signal; $\tau$ is the average propagation delay time; $h(t)$ is the combined Rayleigh and lognormal process, given by $h(t) = R(t)L(t)$; $L(t)$ is a lognormal process governed by parameters $u_i$ and $\sigma_i$, representing the fading envelopes of the shadowing component; $R(t)$ is a Rayleigh process governed by parameter $\sigma^2_r$, representing the fading envelopes of the fast fading component; $\theta(t)$ is a stochastic process with a uniform distributions in the interval $[0,2\pi]$, and $w(t)$ is a zero-mean complex Gaussian background noise with parameter $\sigma^2_w$. The received signals in a narrowband propagation channel are assumed to be associated with complex random process, $Z(t)$, and

$$
Z(t) = a(t) + j b(t) = h(t)e^{j\theta(t)}. \qquad(15)
$$

Thus, the received fading envelope in a narrowband propagation channel at certain time interval can be expressed as

$$
|Z| = r = RL \qquad(16)
$$

The average BER for transmission in a narrowband propagation channel can be written as

$$
P_e = \int_0^\infty P(e\mid r)f(r)\,dr
$$

$$
= \int_0^\infty \left[ \int_0^\infty P(e\mid r)f_{x_1}(r)\,dr \right]f_x(L)\,dL \qquad(20)
$$

where $P(e\mid r)$ is the BER in AWGN channel for an arbitrary modulation scheme at a certain value of $r$ and $f(r)$, $f_{x_1}(r)$,
and \( f_i(L) \) are given by (17), (18), and (19). The inner integral (i.e. \( P(e | L) \)) of Eq. (20) represents the average BER in the presence of Rayleigh fading only. In our case, we first analyze the error performance of the inner integral part of Eq. (20) in the AM system. And then, the Hermite polynomial method [14] is applied to compute the final average BER performance over Rayleigh-Lognormal fading channel in M-ary PSK AM system.

Firstly, the inner integral part of Eq. (20) can be transformed into the function of \( \Gamma \), it is convenient for analyzing the BER performance in M-ary PSK AM system. Therefore, the average and instantaneous SNR given on a certain value of \( L \) is defined as

\[
\Gamma = \mathbb{E}[R^2L^2/2] = \frac{P_S}{P_w} = \frac{L'\sigma_0^2}{2P_w} \tag{21}
\]

and

\[
\gamma = \frac{L'R^2P_s}{2P_w}. \tag{22}
\]

The mixture pdf of received signal envelope in (17) can be simplified by changing to be function of \( \Gamma \) yielding

\[
f_{\gamma}(\gamma) = \int \frac{1}{L'} \exp \left[-\gamma \frac{L'}{L'}\right] \frac{1}{2\pi h \sigma_i} \frac{1}{\Gamma} \left[ \frac{1}{2} \ln(hL_i) - \frac{h\sigma_i}{2h} \right] dL' \tag{23}
\]

Similarly, the mixture cdf of received signal envelope can be calculated as an integral of the Eq. (23)

\[
F_{\gamma}(\gamma) = \int_{-\infty}^{\gamma} f_{\gamma}(\gamma) \, d\gamma
\]

\[
= \left[1 - \exp \left(-\gamma \frac{L'}{L'}\right)\right] \frac{1}{2\pi h \sigma_i} \frac{1}{\Gamma} \left[ \frac{1}{2} \ln(hL_i) - \frac{h\sigma_i}{2h} \right] dL'. \tag{24}
\]

Therefore, the average data throughput \( D(\Gamma, A) \) in term of BPS can be described as

\[
D(\Gamma, A) = \sum_{i=1}^{4} \log_2(M_A) \left\{ \exp \left[-\frac{A_i}{\Gamma} \right] - \exp \left[-\frac{A_{i+1}}{\Gamma} \right] \right\} \times \frac{1}{\sqrt{2\pi h \sigma_i}} \exp \left[-\frac{1}{2} \left( \ln(hL_i) - \frac{h\sigma_i}{2h} \right)^2 \right] dL'. \tag{25}
\]

By substituting Eq. (24) into Eq. (5) and Eq. (6), the average BER for the mode \( i \) in the presence of Rayleigh fading only can be expressed as

\[
P_{d,\gamma}(\gamma) = \frac{1}{\Gamma} \int_{0}^{\gamma} F_{\gamma}(A_i) Q\left[\sqrt{A_{i+1}} \delta_i \right] - F_{\gamma}(A_i) Q\left[\sqrt{A_i} \delta_i \right] + \xi \]}

where

\[
F_{\gamma}(\gamma) = 1 - \exp \left[-\gamma \frac{L'}{L'}\right] \tag{27}
\]

and

\[
\xi = -\frac{\delta \Gamma L}{\sqrt{L'+\delta \Gamma L}} \left[ Q \left[\sqrt{\Gamma \left(A_i+\delta \Gamma L\right)}\right] - Q \left[-\sqrt{\Gamma A_i}\right] \right]. \tag{28}
\]

Therefore, the average BER of a Gray-coded M-ary PSK AM system at specific mode \( i \) can be rewritten as

\[
P_{d,\gamma}(\gamma) = \frac{1}{\Gamma} \int_{0}^{\gamma} F_{\gamma}(A_i) Q\left[\sqrt{A_{i+1}} \delta_i \right] - F_{\gamma}(A_i) Q\left[\sqrt{A_i} \delta_i \right] + \xi \]}

Finally, the Eq. (29) is converted into following form which can be computed by applying the Hermite polynomial method

\[
\int_{-\infty}^{\infty} f(y) e^{-y^2} dy = \sum_{i=1}^{8} w_i f(y) + R_4 \tag{30}
\]

where \( y \) and \( w \) are the abscissas and weight factor for a 12-point Hermit Quadrature [14, Table E.1]. The remainder \( R_4 \) of Eq. (30) is defined by Hermite polynomial method in the equation (E.2). Therefore, the analytical expressions are provided for the average BER performance of AM, used in Eqs. (26)-(30), for the Rayleigh-Lognormal fading channel.

IV. SIMULATION AND NUMERICAL RESULTS

Fig. 2 shows the pdfs of the fading envelopes for three different values of standard deviation parameters \( \sigma_i = 0dB, 4dB, \) and \( 6dB \) and same mean value \( u_i = 0dB \) of the shadowing components. According those three kinds pdfs, the performances of the M-ary PSK AM system is investigated, using the optimum selection approach optimal of Section II-B over Rayleigh-Lognormal fading channel. Assuming that five different M-ary PSK signal constellations corresponding to 4-PSK, 8-PSK, 16-PSK, 32-PSK, and 64-PSK are available at the transmitter. The no transmission (No_Tx) is assigned if the instantaneous SNR is too poor to satisfy the average BER constraint. A six-mode AM is presented to illustrate the effect of standard deviation (\( \sigma_i \)) of various shadowing fading.

![Fig. 2 Probability density function of fading envelope for different channel conditions](image-url)
In order to assess the validation of our numerical analysis and simulation system, the Gray-coded BERs of M-ary PSK modulation schemes are evaluated with numerical and simulated results at the various shadowing fading effects. Fig. 3 show the Gray-coded BERs of M-ary PSK modulation schemes obtained with both the numerical and simulated results, for the $\sigma_z = 0\text{dB}, 4\text{dB}, \text{and } 6\text{dB}$. The simulated Gray-coded BERs of M-ary PSK modulation schemes are calculated using $10^5$ bits, and are averaged over 20 simulation runs. It can be seen that the Gray-coded BERs for the numerical and simulated results are reasonably agree with each other.

The optimal set of SNR thresholds for the six-mode AM system with constant transmitted power when the required BER $P_{th}$ is $10^{-3}$, the standard deviations $\sigma_z$ are $0\text{dB}$, $4\text{dB}$, and $6\text{dB}$, can be seen from Fig. 4. The data throughput performance is evaluated for the three shadowing conditions using different selected modes adaptive modulation according to the Eq.1. The 2-mode AM system is corresponds to choose the “No-Tx” and “4-PSK” modes according to the first SNR threshold. The 3-mode AM system is corresponds to choose the “No-Tx”, “4-PSK”, and “8-PSK” modes according to the first and second SNR thresholds. For this reason, the 6-mode AM system is corresponds to choose the “No-Tx”, “4-PSK”, “8-PSK”, “16-PSK”, and “12-PSK” modes according to the set of SNR thresholds. Furthermore, the Gray-coded BERs of an M-ary PSK AM system are evaluated with numerical and simulated results, for $\sigma_z = 0\text{dB}$, $4\text{dB}$, and $6\text{dB}$, respectively. Figure 5 shows the results. The BER performance of the 3-mode AM system outperform slightly than 2-mode AM system for three kinds situations, according to the optimal set of SNR thresholds. The simulated results indicate that the AM system is capable of satisfying the BER requirement over Rayleigh-Lognormal fading channel.

V. CONCLUSION

In this paper, a close-form expression of the average BER and the average throughput of an M-ary PSK adaptive modulation system with constant transmit power over arbitrarily fading channel is developed. Besides, a simple model adaptation method is applied to derive the set of the optimal thresholds of the partition.
According to the above results, the analytical expressions of the average BER and average data throughput derived for applying AM scheme in Rayleigh-Lognormal fading channel is presented. The effect of shadowing on BER and throughput is also investigated using average BER constraint. Compare to the Mote Carlo simulation method, our analyzed results show good agreement to the simulation ones. The proposed method can be useful in enhancing the data throughput of the communication systems, such as third generation wireless communication systems, where a huge amount of bandwidth is needed for high-speed multimedia services.

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